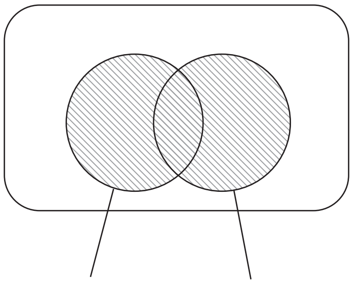
Solution Manual

CHAPTER 1

1.1 Using the same Venn diagram for illustration, we want the probability of outcomes from the two events that lead to the cross-hatched area shown below:



This represents getting A in event 1 and not B in event 2, plus not getting A in event 1 but getting B in event 2 (these two are the common "or but not both" combination calculated in Problem 1.2) plus getting A in event 1 and B in event 2.

1.2 First the formula will be derived using equations, and then Venn diagrams will be compared with the steps in the equation. In terms of formulas and probabilities, there are two ways that the desired pair of outcomes can come about. One way is that we could get A on the first event and not B on the second ($A_1 \cap (\sim B_2)$). The probability of this is taken as the simple product, since events 1 and 2 are independent:

$$\begin{aligned} p_{A_1 \cap (\sim B_2)} &= p_A \times p_{\sim B} \\ &= p_A \times (1 - p_B) \\ &= p_A - p_A p_B \end{aligned} \tag{A.1.1}$$

The second way is that we could not get A on the first event and we could get B on the second ($(\sim A_1) \cap B_2$), with probability

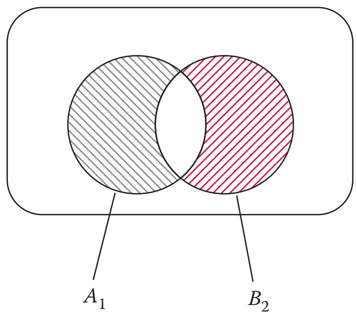
$$\begin{aligned} p_{(\sim A_1) \cap B_2} &= p_{\sim A} \times p_B \\ &= (1 - p_A) \times p_B \\ &= p_B - p_A p_B \end{aligned} \tag{A.1.2}$$

Since either one will work, we want the or combination. Because the two ways are mutually exclusive (having both would mean both A and $\sim A$ in the first outcome, and with equal impossibility, both B and $\sim B$), this or combination is equal to the union $\{A_1 \cap (\sim B_2)\} \cup \{(\sim A_1) \cap B_2\}$, and its probability is simply the sum of the probability of the two separate ways above (Equations A.1.1 and A.1.2):

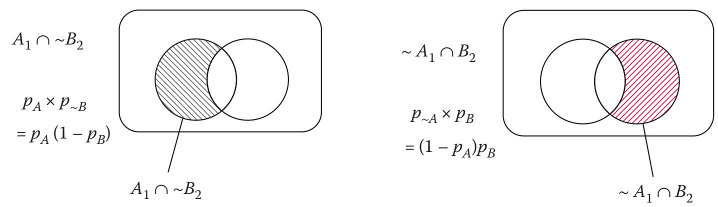
$$\begin{aligned} p_{\{A_1 \cap (\sim B_2)\} \cup \{(\sim A_1) \cap B_2\}} &= p_{A_1 \cap (\sim B_2)} + p_{(\sim A_1) \cap B_2} \\ &= p_A - p_A p_B + p_B - p_A p_B \\ &= p_A + p_B - 2 p_A p_B \end{aligned}$$

The connection to Venn diagrams is shown below. In this exercise we will work backward from the combination of outcomes we seek to the individual outcomes. The probability we are after is for the cross-hatched area below.

$$\{A_1 \cap (\sim B_2)\} \cup \{(\sim A_1) \cap B_2\}$$



As indicated, the circles correspond to getting the outcome A in event 1 (left) and outcome B in event 2. Even though the events are identical, the Venn diagram is constructed so that there is some overlap between these two (which we don't want to include in our "or but not both" combination. As described above, the two cross-hatched areas above don't overlap, thus the probability of their union is the simple sum of the two separate areas given below.



Adding these two probabilities gives the full "or but not both" expression above. The only thing remaining is to show that the probability of each of the crescents is equal to the product of the probabilities as shown in the top diagram. This will only be done for one of the two crescents, since the other follows in an exactly analogous way. Focusing on the gray crescent above, it represents the A outcomes of event 1 and not the B outcomes in event 2. Each of these outcomes is shown below: